A Tale Of 2 Rabbits

This document is located at http://lasttechage.wordpress.com/pdf-references/

Exponential growth

This is an example of how things grow when the process is unrestrained by outside influences … A.K.A. *unconstrained*, *initiation*, or *exponential* growth. In pop culture, *exponential* seems to mean something like "Awesome!" or "Golly that's big!" It actually a technical word for a specific process that builds slowly at first, then expands at nearly inconceivable rate.

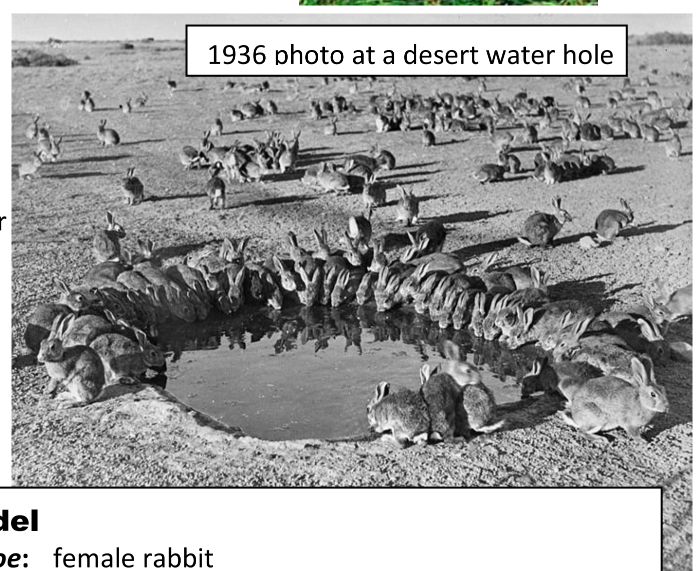
What it means in math

The word indicates an algebraic function of form: constant to a variable power.

Examples would be 2^x or $10^{t/T_{10}}$ or e^{t/T_e}. The last constant, e, is the irrational number 2.71828… This pops up whenever you look at a process that grows continuously (not in steps like a bank account)

A Tale of 2 Rabbits

Australia and rabbits have been joined since the mid 1800's. Rabbits were introduced in the 1700's but the population exploded onto the landscape only after a new hybrid of gray rabbits was released in 1859. Details differ depending on the source, but an estate owner, Thomas Austin, released a small group as game animals. In the Wikipedia account (bunnies), 12 rabbits were released in October 1859. *Some references cite 24 rabbits released around Christmas in 1859.* At the time, Austin said: "The introduction of a few rabbits could do little harm, … provide a touch of home, … [and] a spot of hunting." 10 years later (1869), the harvest of 2 million rabbits had virtually no impact on the then-current population.



1936 photo at a desert water hole

We model the growth of rabbit populations,
but be warned:

This could be the back story for a new Stephen King novel or a gothic horror film featuring cute little furry creatures that are actually horrid gremlins, living a life from the outer circles of Dante's vision of Hell.

Data for our model

***Doe*:**	female rabbit
***Buck*:**	male rabbit
***Kit*:**	baby rabbit
Breeding Season:	8 months: mid Feb – mid Oct
***Take*:**	Successful impregnation
Obligate Conception:	Doe ovulates soon ***after*** insemination
Gestation:	30-32 d.
Maturation:	5-6 months birth-to-sexual maturity
Litter size:	4–8. observed range: 2–12.
birthing to next take:	Doe available within hours, Take in 30 – 45 days
take-next take:	Approx 3 mo. (gestation + recovery)

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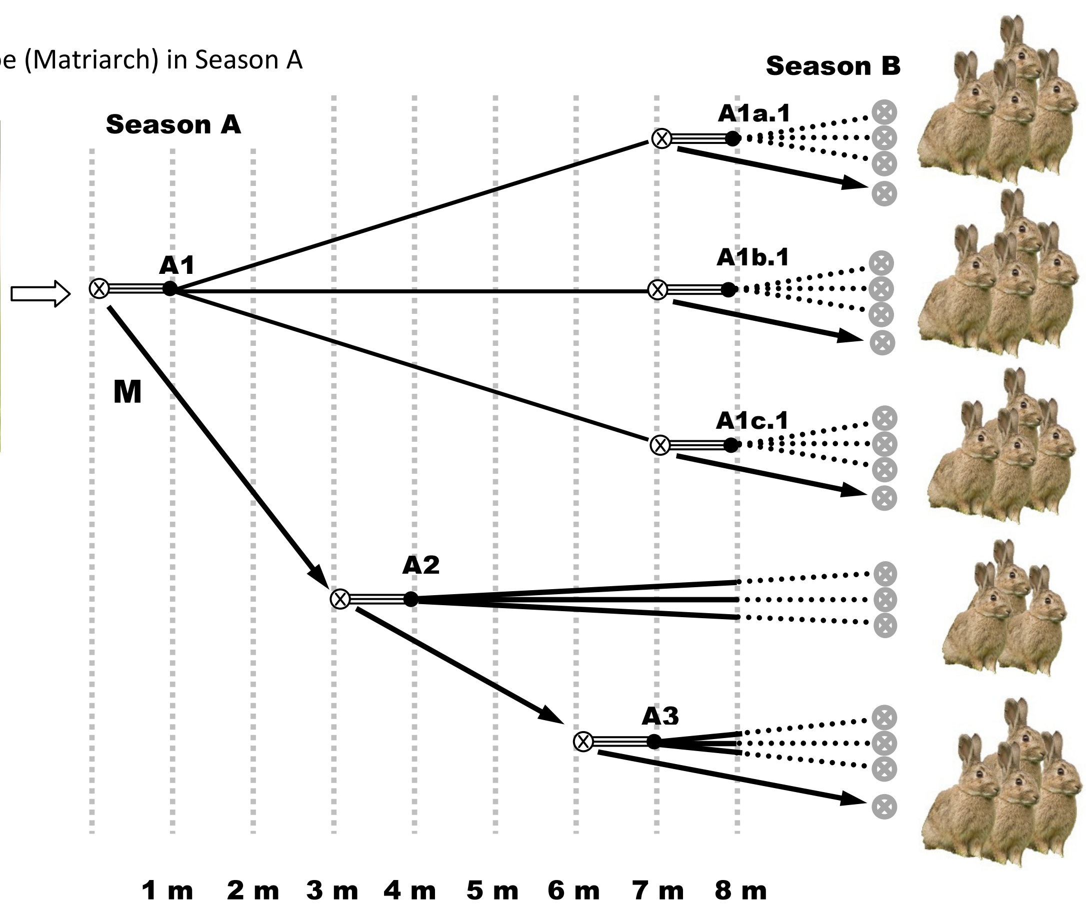
Values and symbols used for this model –

Breeding Season:	8 months (might be a bit short)
Birth ●	30-32d. Average = 1 mo
Take ⊗:	Successful impregnation
●——⊗:	maturation time average of 6 mo
Litter size:	6 Rather variable, young does fewer, larger litters with age
Doe per litter:	3 assume 50-50 doe/buck
Take-next-Take ⊗---⊗:	3 months (1mo gestation + 1 or 2 mo recovery) … this may be slightly long estimate

Analysis The number of bucks is not important, population growth is controlled by the doe count. Use data to diagram female kits from one doe through one breeding season. Assume that the rate of reproduction depends only on rabbit physiology, with no reductions due to outside influences (unconstrained growth assumption).

Start with single doe (Matriarch) in Season A

Matriarch



Count the rabbits:

M	+ A1	+ A1a.1	+ A1b.1	+ A1c.1	+ A2	+ A3	
1	+ 3	+ 3	+ 3	+ 3	+ 3	+ 3	= 19

Total at end of season A: 19.

This actually means 38 rabbits in total, from the single doe at the start (along with her buck).

If there are no external reduction of population members, then, in the second breeding season, each of the 19 does from season A will generate 19 females on the ground at season B ($19 \times 19 = 361$). In season C, each of the 361 breeds 19 more ($19^3 = 6\,859$), and so forth, season after season after season.

The model —
Number D of does at <u>end</u> of breeding season N: $D = 19^N$

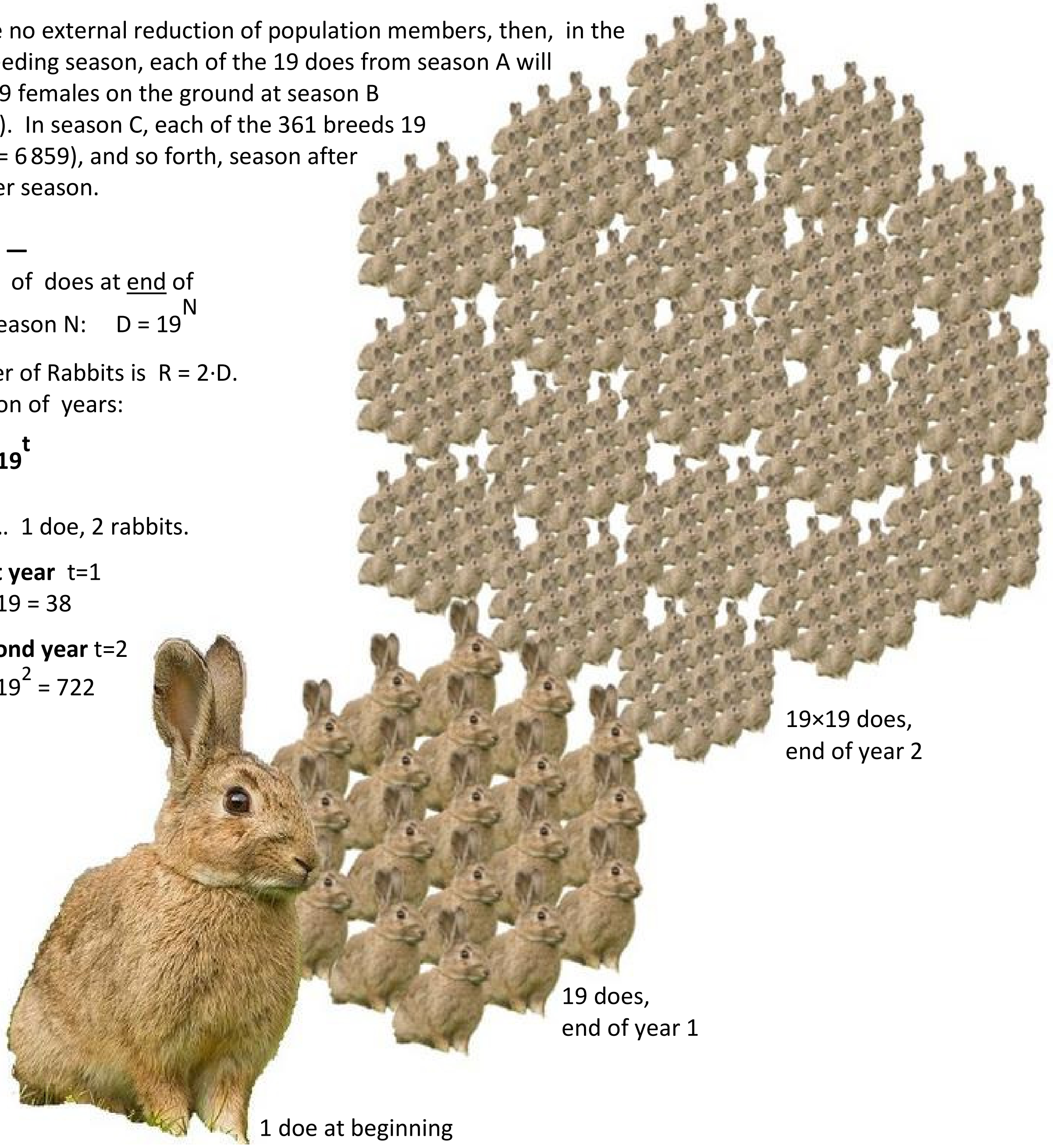
The number of Rabbits is $R = 2 \cdot D$.
As a function of years:

$$\mathbf{R = 2 \cdot 19^t}$$

Start: t=0 … 1 doe, 2 rabbits.

End of first year t=1
$R = 2 \cdot 19 = 38$

End of second year t=2
$R = 2 \cdot 19^2 = 722$



Assuming no external influences, such as predators, diseases, or lack of food or water, by the end of the 5th breeding season (year 5) there will be $R = 2 \cdot 19^5$ = 5 million bunnies hopping about … all from the one matriarch doe and any number of helpful bucks.

Can exponential growth be maintained? Consider the time to form an *Australian bunny carpet*… Suppose Australia (7.6 million km^2) were solidly covered by rabbits, nose to tail. A rabbit needs about 18×8 inches (930 cm^2), so a continental carpet would need 82 trillion (82×10^{12}) rabbits. Unconstrained base-19 growth would reach this impossible value in less than 10.7 years. ($82 \times 10^{12} = 2 \times 19^t$)

No wonder that when Australians harvested two million rabbits after the first 10 years, it made no noticeable dent in the population. Had there been unconstrained free growth for that entire decade, Australia would have had a true carpet of furry bunnies.

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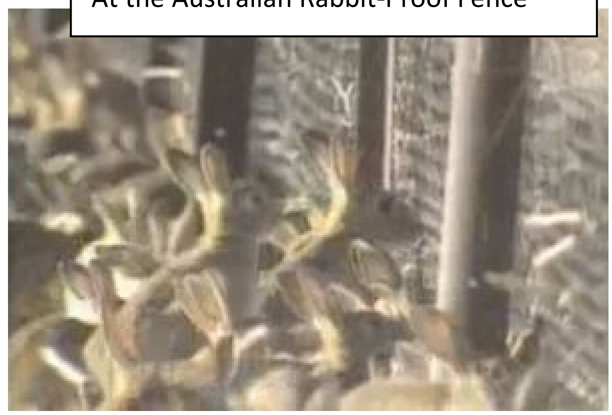
At the Australian Rabbit-Proof Fence



During the first several years after release, the rabbits must have thought they were in heaven. They had ample resources and few predators. This must have certainly have ended by the fourth year.

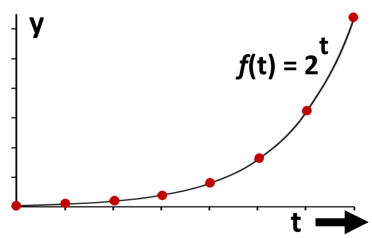
The monstrous biological growth rate means that the rabbit population must have formed as an expanding mass with free growth at the edges, but being *highly* constrained within. Starvation, thirst, diseases, along with indigenous predators must have severely limited the central colony. Once the rabbit colony reached the limits of its possible growth, the entire population existed in this *constrained* state. This species seems to thrive in conditions that are miserable to its individuals.

This is a clear example of exponential growth –relatively slow at the start, then nearly unbelievable jumps in population size. Growth like this can never be sustained after the species establishes itself in its environment.

Exponential growth:

The shape:

Typical shape for base 2 growth



The process:

To get the characteristic shape,

(A) The amount of new growth next cycle will be proportional to how much is currently there.

(B) The system's growth rates should not be influenced by happenings in the external world.

The algebra:

Growth is in cycles per year, the quantity Q will be (such as investing at compound interest rate)

$Q(t) = Q_0 (1+k/m)^{mt}$

t is the elapsed time (years, seconds, etc)
Q_0 is quantity at start (t = 0)
Q(t) is quantity at any time thereafter
m is the number of cycles / time
k is the growth rate (4% means k = 0.04)

Growth is continuous, the quantity Q will be (such as using up oil from a reservoir)

$Q(t) = Q_0 b^{kt}$

t is the elapsed time
Q_0 is quantity at start (t = 0)
Q(t) is quantity at any time thereafter
b is base constant
k growth rate $k = 1/T_b$

T_b is time for Q to grow by a *factor* of b.

that is : when $t = \mathbf{T_b}$, $Q(t) = b\,Q_0$