Visualize Exponential Growth

This document is located at http://lasttechage.wordpress.com/pdf-references/

1955	2.9
1956	3.2
1957	3.3
1958	3.2
1959	3.4

Fig. 1: Raw data

Here Is An Example Consider the amount of copper mined in the world between 1900 and 2009. Fig 1 is a snippet of an Excel database from the U.S. Geological Survey. http://minerals.usgs.gov/ds/2005/140/#copper.

Fig 1: Mining production data shown in successive rows as | Year date | Million tons |

Fig 2: Plot these data in a standard Cartesian linear graph. Here, we use Excel's charting capability to do so. The blue dots are the raw data points, the blue line joins the points so you can see the trends more easily.

The dots appear to scatter about an (imagined) curved line that might be exponential due to the upwards curvature. (Read next page to see why we expect this.)

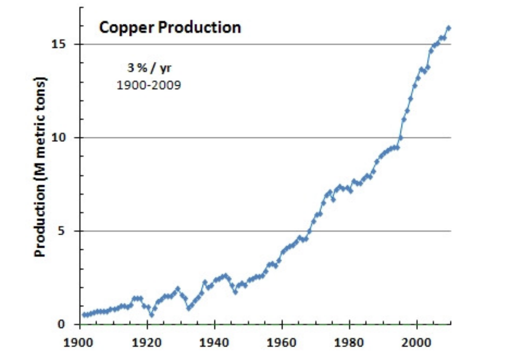

Fig 2: Copper production data: linear-linear

Test for exponential growth: Plot the logarithms of the Production data [log(y) values vs. date] rather than the raw data itself.

Take the log of the values (units: million tons): Log(2.9) = 0.462. Log(3.2) = 0.477, Log(3.3) = 0.491 … and on through the table.

This is tedious even with a hand calculator. Instead, Make an Excel Chart, click the right axis, right-click & select [Format Axis], click [Logarithmic scale] ⊙. Result is Figure 3.

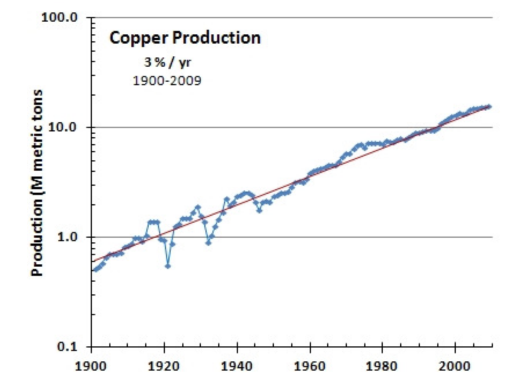

Fig 3: Copper production data: log-linear

Fig 3 is the same as Fig 2, except the log of the data is graphed, but the values along the Y axis corresponds to the original numbers, not their log values. This is called a "log-linear" or "semi-log" plot. The straight (red) line through the data is the production function P(t) used to model the data.

$$P(t) = 3.3\, e^{0.03(t-1957)} = P_0\, e^{r(t-t_0)}$$

- P_0 = 3.3 million tons
- t_0 = 1957 … when P_0 occurred
- r = 0.03 growth/year (+3%/year)

Last check: you can see the data follow an exponential model curve with a 3% growth rate with a linear chart of P as the thin red line on the same graph as the data. Since data points

[A}vary about the red "trendline" and
[B] the trendline is linear on a semilog plot,

we conclude we are observing exponential growth for the drainage of copper from its underground reservoirs.

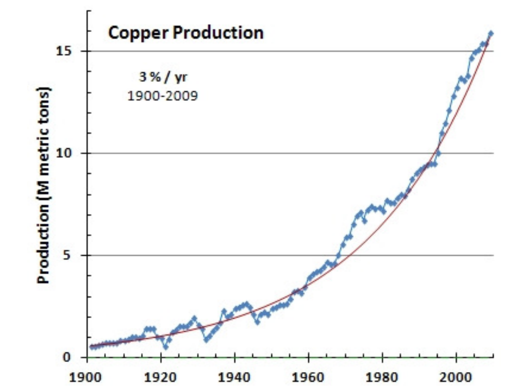

Fig 4: Copper production data, linear scales

Fig. 4 is a linear-linear replot of Fig2, displaying the exponential trendline from Fig 3. (The math 'least squares best fit' test gives r=0.0308/yr)

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Algebra

To understand what we are about to do, you need the Algebra-II section on exponentials and logarithms.

Suppose you have modeled a growing function (like the copper growth, last page) as an exponential. Nature uses the Euler constant e as the fundamental base for continuous processes (not in steps or jerks). It would be easiest to model with the e base

Write the model curve as an exponential fuction using nature's own base, e ($\approx$ 2.71828183) $P(t) = P_0 e^{r(t-t_0)}$

Graph the function:

	$P(t)$	$= P_0 e^{r(t-t_0)}$
Use log rules	$\ln(P/P_0)$	$= r\cdot(t-t_0)$
Change symbols used	y	$= \ln(P/P_0)$
Linear relation ($y = m\,t + b$)	y	$= r\cdot(t - t_0)$ ... *slope is* r

Plot y vs. t this on a graph (that is, plot $\ln(P/P_0)$ vs. t), get a straight line with slope r. Mark the Y axis with steps in values of P, not steps in values of ln(P). This is a log-linear "semilog" plot. 'Common' base-10 logs could be used.

Exponential function

$P(t) = P_0 e^{r(t-t_0)}$

In the natural base e

P_0 = starting amount

t_0 = time when A_0 occurred

because $e^0 = 1$

r = natural growth rate (1/time)

time to grow factor of e is 1/r

In base 10, the *common* base

$P(t) = P_0 10^{r\cdot\log(e)\cdot(t-t_0)}$

$= P_0 10^{k\cdot(t-t_0)}$

t_0 = time when P_0 occurred

because $10^0 = 1$

k = base-10 growth (1/time)

time to grow factor of 10 is 1/k

The Extraction Process

Initial exponential growth. Most reservoir usage processes grow exponentially, because external demand absorbs all that can be produced; there are no outside constraining influences . That is, growth is controlled by the details of the process.

Processes near peak extraction. External brakes on growth will almost certainly have happened before the reservoir has been half emptied. When external constraints happen, the growth becomes much slower than exponential. In fact, in many cases the initial growth rate is the largest to be seen at the reservoir. The rate value can drop a lower exponential value as the fill level drops to the halfway point.

Near the peak in extracted amount. As the reservoir level falls, the extraction becomes successively more difficult. The *Hubbert model* of reservoir usage predicts that the maximum amount that can be removed per year is reached just about the time that half of the reserve has been used.

After peak production. There is no law of nature requiring any particular pattern about the production roll-over point, the peak value. The production decay rates after the peak do not have to match original initiation rates, either. Peak and end-period extraction from the reservoir will be controlled by internal and external circumstances (such as sky-rocketing prices and new and more exotic extraction techniques).

Does the data have segments with exponential growth?
Make a semi-log graph of a data set and look for straight lines.