Suddenness of exponential growth hard to grasp

The Chess Board's Tale

Kingdom is ravaged by terrible beast
- destroys crops, children,
- villagers, taxpayers.

King sends out plea for help
Brave Young Man responds

KING: Save us, I will give anything in reward
BYM: Ok, my reward will by your daughter’s hand in marriage
KING: Yes, yes, anything

Month later, Kingdom saved! Victorious BYM returns bloody,
limping, arm in sling ... ready for his new wife

King reneges: About that daughter thing, sorry, Dude!
Changed my mind – Rich old count down the way just offered great dowry

BYM: Well, ok, I withdraw my claim if you do this...
  Day 1: 1 penny on square 1 of chess board,
  Day 2: 2 pennies on second square,
  Day 3: 4 pennies on third,
  Each successive day, 2× previous day.

Ancient story
Each square, 2× the number of pennies

King cogitates

"Hmm —
4th day, only 1+2+4+8 = 15 pennies
Square 8... "I can count that" 128
Square 16 ... +8 squares 8*128 1024
Square 24 probably 8*1024 8192
Square 64 ... 268,435,456 ¢
only ≈ $2.70 M?
"Is BYM stupid, or ...
could I have made a mistake?"

Actually, our King is even worse with math than he is with running his kingdom!
The King saw process:

• Tomorrow's gain in height is always proportional to the stack's height today.

**Tomorrow** — the stack will be twice as tall as it is today.

Hey! No problem.

*Tomorrow:* Only $128¢ 7"

*Day after?*  $256¢ 14"

• The royal treasury has truly huge resources. Tomorrow's demand will be trivial to meet, as well as the next day's!

"This is easy, surely we can keep going *forever.*"
On day 8, 128 pennies were placed on square 8

If you double each step — square 16 will be 150 feet high! Really?
King needed expert help

Call in ... the Court Mathematician

The CM ponders, measures, calculates a penny 1.4 mm thick (0.055”), … daily doubling ... By Day 22 he was ready to explain here is his final diagram The game had reached square 22 (m) the stack towered more than 1 mile with \(2^{21} = 2,097,152\) coins.

Tomorrow, the coin stack will be twice as high.

No outside constraints to growth:
CM: "M'lord, let us assume that progress is limited only by the game rules and we can, build a new tower each and every day. Here is what will happen …"
Day 29  Enjoyn' a quiet orbit about the Station. Suddenly a coppery spike shoots from Earth and zips past.

*Tomorrow's stack will go twice as far.*
Day 39 Imagine the surprise at our Lunar Colony base as the penny spike shoots past the moon!

*Tomorrow's stack will go twice as far.*
Day 40  The pennies in the stack out number the twinkling stars in the Milky Way (200-400×10⁹)

*Tomorrow's stack will go twice as far.*
Day 48 Length of the penny stack exceeds Earth-Sun distance (93 M mi, 150 M km).

*Tomorrow > diameter of the Earth's orbit*

Tomorrow's stack will go twice as far.
The CM's story – 5

Day 54  Penny stack now longer than the *radius* of the solar system. 7.5 billion miles or $12 \times 10^9$ km

*Tomorrow's stack exceeds diameter of solar system.*

*Tomorrow's stack will always go twice as far.*
CM: "true, sustainability might become an issue? but what if we continued ..."

to the very end

Day 64 Stack > 1 light year.

and just 2 days longer

Day 66 ...at Alpha Centauri

Each and every inevitable day, exponential growth in the penny stack

Tomorrow's stack of pennies must always go twice as far as today's
Aside from inventing warp drive ...

▶ **The King clearly had a couple bad months**

Several endings:

- King gives in to the inevitable – BYM, Princess marries
- King is full of rage – orders BYM executed (with consequences)
- ... there are other endings, too.

▶ **This is story of unconstrained 'explosive' growth**

- Coin stack starts slowly, no problem – grows unbelievably fast
- Rapid growth – poses impossible logistics problem (physics truths aside)
  - Unbelievable challenge to ever assemble daily coin stack
  - Impossible growth demand on copper supply-line
  - Relentless expansion of facility needs

**Most growth** begins with free, unconstrained expansion
winds up in non-exponential *highly constrained* situation.